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Multi-Period Portfolio Optimization Under Uncertainty using Diversification Measure

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ABSTRACT

Modern Portfolio Theory, with the introduction of the mean-variance model, established the first theoretical framework for portfolio selection. While the mean-variance model is widely used as the foundation for a broad range of problems in this field, it is criticized by researchers for its high sensitivity to input parameters and its tendency to select concentrated portfolios. To address these shortcomings, this research introduces the Robust Mean-Variance Entropy model. The Robust Mean-Variance Entropy model seeks to control the risk arising from estimation error by employing robust optimization. Furthermore, by incorporating Yager's entropy as a diversification measure, it aims to increase the diversification of the optimal portfolio by preventing concentrated allocations. The proposed model, which has a multi-period structure, was studied over an 18-month period, and its performance was evaluated and validated using historical data from the top 20 stocks of the S&P 500 index. When compared to an equally weighted portfolio, the results of the Robust Mean-Variance Entropy model show that while achieving high returns that were very close to its counterpart, the model demonstrated impressive performance in terms of risk management, effectively protecting its underlying capital during market downturns.

Keywords: Portfolio Optimization, Robust Optimization, Diversification Measure, Multi-Period Approach

Introduction

The theory and practice of portfolio optimization have undergone profound transformations since the seminal introduction of Modern Portfolio Theory (MPT) by Markowitz, which formalized the mean–variance paradigm as a rational framework for balancing expected return and risk (1). Despite its enduring influence, the classical mean–variance model has been persistently criticized for its extreme sensitivity to input parameters, especially expected returns and covariances, leading to unstable and highly concentrated portfolios that perform poorly out-of-sample (2). This phenomenon, often described as "error maximization," emerges because small perturbations in estimated means can produce disproportionately large changes in optimal weights, thereby undermining the reliability of the solution (3). Empirical research has shown that naïve diversification strategies such as the equally weighted 1/N portfolio frequently outperform optimized portfolios when estimation error is substantial, raising serious questions about the practical superiority of sophisticated optimization models (2). These concerns have motivated a rich stream of research devoted to enhancing robustness, improving diversification, and extending portfolio models to more realistic multi-period settings. In parallel, the rapid growth of financial data, algorithmic trading, and AI-based analytics has intensified the need for models that integrate predictive intelligence with structural resilience (4).

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Consequently, contemporary portfolio research increasingly seeks to reconcile three objectives: statistical robustness, diversification enhancement, and dynamic adaptability under uncertainty.

A central response to estimation risk has been the development of robust optimization frameworks. The foundational work of Ben-Tal and Nemirovski introduced robust counterparts to linear programming problems contaminated with uncertain data, providing tractable solutions that remain feasible across predefined uncertainty sets (5). Subsequent theoretical advancements established robust optimization as a comprehensive methodology for decision-making under parameter ambiguity, emphasizing its computational tractability and structural elegance (6). In portfolio contexts, robust approaches modify the mean–variance framework by allowing expected returns or covariance parameters to vary within uncertainty sets, thereby protecting the solution against adverse deviations (3). The “price of robustness” concept formalizes the trade-off between performance and protection, demonstrating that moderate conservatism can significantly stabilize portfolio allocations without excessive sacrifice of expected return (3). Robust multiperiod portfolio management models further incorporate transaction costs and temporal dependencies, illustrating how dynamic robust allocation strategies outperform static ones in volatile environments (7). These theoretical contributions have been complemented by empirical applications in emerging and small markets, where robust maximum diversification strategies have shown superior resilience under thin liquidity and structural constraints (8). Collectively, the robust optimization literature provides a mathematically rigorous foundation for mitigating estimation risk, yet it does not automatically guarantee diversified allocations unless explicit diversification measures are embedded in the objective structure.

Diversification, traditionally interpreted as the reduction of unsystematic risk through asset spreading, has increasingly been operationalized using entropy-based measures and alternative concentration metrics. Shannon entropy and related information-theoretic constructs have been employed to penalize weight concentration and encourage uniform capital distribution (9, 10). However, entropy-based diversification must be carefully balanced against risk-return objectives to avoid mechanical equal weighting. Recent research has highlighted the importance of integrating diversification measures directly into optimization formulations to prevent corner solutions (11). In small and frontier markets, diversification constraints significantly improve stability and risk-adjusted performance, especially when combined with robust return modeling (8). Furthermore, behavioral finance research suggests that regret aversion and other cognitive biases influence portfolio concentration tendencies, underscoring the need for structural diversification mechanisms that counteract human overconfidence and herding effects (12, 13). In cryptocurrency markets, credibilistic CVaR-based allocation under practical constraints demonstrates that diversification remains crucial even in highly volatile digital asset classes (14). These developments indicate that diversification is not merely a heuristic principle but a quantifiable design objective requiring explicit modeling attention within optimization frameworks.

Parallel to robustness and diversification advances, multi-period portfolio optimization has gained renewed prominence due to the dynamic nature of financial markets. Early recursive formulations and dynamic programming approaches laid the groundwork for multiperiod extensions of the mean–variance paradigm (7). More recently, bibliometric analyses reveal a rapid expansion of multiperiod portfolio research, emphasizing scenario-based modeling, stochastic programming, and practical constraints integration (15). Real-world portfolio management involves transaction costs, rebalancing policies, and budget uncertainty, all of which necessitate multistage modeling frameworks (16, 17). Scenario-based planning under budget uncertainty illustrates how multistage optimization improves allocation stability in complex logistical and financial systems (17). Moreover, regret-based stochastic portfolio models demonstrate that volatility-sensitive risk measures can be embedded within dynamic structures to enhance adaptability (18). These approaches collectively emphasize that static single-period optimization inadequately reflects practical investment realities, where portfolios are continually revised in response to new information and evolving risk conditions.

The integration of artificial intelligence and machine learning has further transformed portfolio optimization. Machine learning algorithms enhance return forecasting accuracy and facilitate risk-adjusted optimization in complex markets such as cryptocurrencies (19). Time-series forecasting combined with machine learning techniques has been shown to improve dynamic asset allocation decisions, particularly when embedded within structured optimization models (20). Reinforcement learning frameworks optimize portfolio selection through iterative stock ranking and adaptive decision rules, bridging predictive analytics and optimization control (21). AI-driven portfolio evaluation frameworks extend beyond traditional variance-based metrics, incorporating alternative performance indicators and nonlinear risk assessments (4). At the same time, ESG constraints have introduced additional structural dimensions to portfolio design, requiring optimization models to balance sustainability objectives with financial efficiency (22). Entropy-based fuzzy optimization models further demonstrate how information uncertainty can be integrated into portfolio selection under ambiguous data environments (10). These technological and methodological innovations underscore a paradigm shift: portfolio optimization is evolving from purely statistical mean–variance trade-offs toward hybrid intelligent systems combining robustness, diversification, and predictive analytics.

Despite these advancements, a conceptual gap persists at the intersection of robust optimization, entropy-based diversification, and multiperiod dynamic modeling. While robust linear programming under uncertainty sets addresses parameter ambiguity (5), and budgeted uncertainty improves tractability (3), few studies systematically integrate entropy-based diversification into a robust multiperiod mean–variance structure. Furthermore, empirical evidence comparing optimized portfolios with naïve benchmarks suggests that structural improvements must translate into tangible out-of-sample resilience to justify model complexity (2). Recent applications across stock markets and alternative asset classes confirm that combining robust risk measures, diversification metrics, and practical constraints enhances stability and risk-adjusted performance (8, 14). However, the literature lacks a unified framework that simultaneously controls estimation error, enforces diversification via entropy, accommodates transaction costs, and operates within a repeated multiperiod structure informed by contemporary predictive insights (4, 19). Addressing this gap requires synthesizing robust optimization theory (6), entropy-based diversification principles (11), multiperiod dynamic modeling (15), and behavioral and practical constraints (12).

Therefore, the aim of this study is to develop and empirically evaluate a robust multi-period mean–variance portfolio optimization model that integrates entropy-based diversification and budgeted uncertainty mechanisms, and to compare its out-of-sample performance against a naïve benchmark under realistic transaction cost conditions.

Methods and Materials

Given the problem definition and solution approach of this study, the research is classified as analytical and developmental. Furthermore, as it utilizes historical data of selected stocks, this research falls into the category of retrospective studies. On the other hand, considering the expected objectives, developing a mathematical model for the portfolio optimization problem and investigating performance improvements and comparing results, this research can be described as outcome-oriented.

The asset universe in this research consists of 20 selected stocks with the highest market capitalization from the S&P 500 index, covering an 18-month period. The selected companies' stocks are from various industrial groups to partially mitigate the impact of unsystematic risk.

To better illustrate the relationships between the stocks, their correlation matrix has been calculated and is displayed in Figure 1. A closer look at Figure 1 reveals a high correlation coefficient among the stocks with the highest market capitalization, indicating a similar pattern of return fluctuations among them.

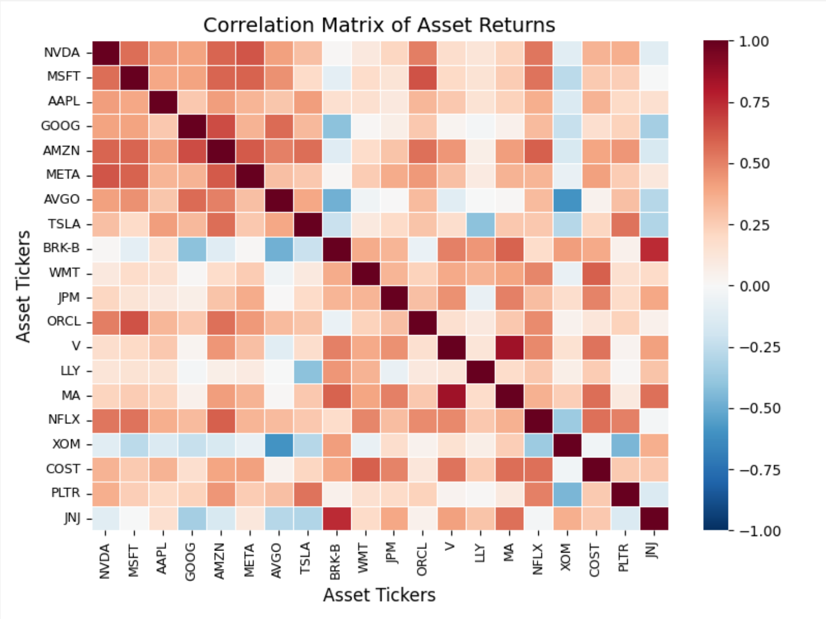

Figure 1. Heatmap Correlation

The mean-variance model is a quadratic multi-objective optimization model comprising two objective functions—return and risk—and a budget constraint. In this model, x i, the decision variable, represents the weight allocated to stock i in the portfolio. Additionally, μ i is the mean return of each stock, and $Cov_{i,j}$ is the covariance between stocks i and j. The basic mean-variance model is as follows:

Maximize:

$$\sum_{i=1}^{n} x_i \mu_i - \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j Cov_{i,j} \quad (1)$$

s.t

$$\sum_{i=1}^{n} x_i = 1 \quad (2)$$

$$x_i \geq 0 \; ; \; \forall i = 1, \ldots, n \quad (3)$$

In Equation 1, the first term calculates the portfolio return, and the second term calculates the portfolio risk. Equation 2, known as the budget constraint, ensures that the sum of allocated weights and participation in the portfolio is at most 100%. Equation 3 indicates the non-negativity requirement for (i.e., no short selling), and n represents the number of stocks. Subsequently, we develop the above basic model by incorporating the considerations mentioned in the research background.

Applying upper and lower bound constraints (Equation 4) to the problem eliminates the possibility of allocating very small values to x_i. Small values of x_i not only have a negligible impact on portfolio performance but also impose higher transaction costs on the model and portfolio.

Additionally, when constructing a portfolio, controlling the permissible number of stocks to be selected is crucial. The more stocks present in the portfolio, the more complex the process of monitoring and evaluating portfolio performance becomes, and the longer the decision-making time for necessary actions. Therefore, a cardinality constraint (Equation 5) is used to define the minimum and maximum allowable number of stocks in the portfolio.

$$L_i y_i \leq x_i \leq U_i y_i \ ; \ \forall i = 1, \dots, n \tag{4}$$

$$C_{Min} \leq \sum_{i=1}^{n} y_i \leq \ C_{Max} \ ; \ \forall i = 1, \dots, n \tag{5}$$

In the above expressions, U_i and L_i represent the maximum and minimum weights allocatable to x_i, respectively; C_{Min} is the minimum number of stocks, and C_{Max} is the maximum number of stocks allowed in the portfolio. Additionally, y_i is a binary variable that equals 1 if a weight is allocated to the i-th stock and 0 otherwise.

To more accurately model real-world problems, the costs of buying and selling each stock, referred to as commissions or transaction costs, can be incorporated into the portfolio optimization problem. A common method for adding a transaction cost function to the model is to deduct a fixed percentage of the total weight allocated to each stock, rather than deducting a monetary cash amount. Assuming the fixed commission percentage is TC, the transaction cost function for stock i is as follows:

$$Transaction\ Costs = \sum_{i=1}^{n} TCx_i \tag{6}$$

Given the points discussed in the research background chapter, if the mean return of any stock deviates from its assumed deterministic value in the base model, the obtained solution may no longer be optimal or even feasible. To address such conditions, the mean return of each stock can be considered an uncertain parameter in the model. In this research, stock returns are considered the uncertain parameters of the problem and are modeled using a budgeted uncertainty set of the form $[\mu_i - d_i, \mu_i + d_i]$. In this set, μ_i is the calculated mean return and d_i is the maximum permissible deviation from the mean.

The primary goal of portfolio selection is to achieve maximum possible return. However, if the performance of one or more stocks does not meet expectations, the entire portfolio must be reviewed. Portfolio review can be conducted at fixed intervals, during which the weights of stocks in the portfolio are adjusted, increased or decreased, according to their performance in each period. To achieve this, the problem model must be extended to a multi-period state by implementing certain modifications and adding relevant and necessary constraints.

Generally, multi-period models involve two common approaches: 1) Repeated Optimization and 2) Portfolio Rebalancing. In the repeated optimization approach, at the beginning of each period, the problem is solved again using new and updated data, without considering the portfolio obtained in the previous period. In this case, there is no guarantee of similarity between the portfolio in each period and the previous one, and not only the allocated weight composition but also the optimal stock composition may change entirely. Additionally, if the changes applied to the optimal portfolio at the beginning of each period are extensive, transaction costs will increase proportionally.

In the second approach, however, the goal is to maintain the proportionality of the weights allocated in the initial period. In other words, as the price of each stock in the portfolio changes, the portfolio's value also changes, causing the percentage participation of each stock to increase or decrease compared to the beginning of the period. To preserve the initial period weights throughout the investment horizon, a portion of the stocks that have appreciated and whose weights in the portfolio have increased must be sold, while a specific percentage of stocks that have experienced price declines must be purchased, so that the portfolio's weight composition remains constant. In this approach, due to generally lower trading volumes, transaction costs are significantly lower compared to the first approach.

In the first approach, since the optimization problem is solved based on the latest available information on each stock's performance, there is an opportunity to improve portfolio performance by considering stocks with higher growth potential. Therefore, in this research, the multi-period structure of the problem is modeled using the first approach, i.e., repeated optimization. Furthermore, the concept of entropy, which was previously examined in detail, will influence portfolio performance within the multi-period model structure.

Given the nonlinear nature of the Yager entropy mathematical relation and the transaction cost function after considering the multi-period structure, we first introduce and rewrite the linear form of each, then formulate the final problem model. The table below shows the linear form of each case along with the necessary auxiliary constraints for linearizing the expressions.

Table 1. Linear vs. Non-Linear formulations in Model

Linearized Formulation	Non-Linear
$\sum_{i=1}^{n} \boldsymbol{TCv_{t,i}}$ *s.t* $\boldsymbol{v_{t,i}} \geq \boldsymbol{x_{t,i}} - \boldsymbol{x_{t-1,i}}$ $\boldsymbol{v_{t,i}} \geq -(\boldsymbol{x_{t,i}} - \boldsymbol{x_{t-1,i}})$ (7)	$\sum_{i=1}^{n} TC\lvert x_{t,i} - x_{t-1,i}\rvert$
$\sum \boldsymbol{s_{t,i}}$ *s.t* $\boldsymbol{s_{t,i}} \geq \boldsymbol{x_{t,i}} - \frac{\boldsymbol{1}}{\boldsymbol{n}}$ $\boldsymbol{s_{t,i}} \geq -(\boldsymbol{x_{t,i}} - \frac{\boldsymbol{1}}{\boldsymbol{n}})$ (9)	$\sum \lvert x_{t,i} - \frac{1}{n}\rvert$ (8)

Minimize:

$$\lambda \sum_{i=1}^{n} \sum_{j=1}^{n} x_{t,i} x_{t,j} Cov_{t,i,j} + (1-\lambda) \sum_{i=1}^{n} s_{t,i} \tag{10}$$

s.t

$$\sum_{i=1}^{n} \mu_{t,i} x_{t,i} - \left(z\Gamma + \sum_{i=1}^{n} p_{t,i} \right) - \sum_{i=1}^{n} TCv_{t,i} \geq \mu_0 \tag{11}$$

$$\sum_{i=1}^{n} x_{t,i} = 1 \tag{12}$$

$$L_i y_{t,i} \leq x_{t,i} \leq U_i y_{t,i} \; ; \forall t \tag{13}$$

$$C_{Min} \leq \sum_{i=1}^{n} y_{t,i} \leq C_{max} \tag{14}$$

$$z + p_{t,i} \geq d_i x_{t,i} \; ; \forall i \tag{15}$$

$$s_{t,i} \geq x_{t,i} - \frac{1}{n} \tag{16}$$

$$s_{t,i} \geq -(x_{t,i} - \frac{1}{n}) \tag{17}$$

$$v_{t,i} \geq x_{t,i} - x_{t-1,i} \tag{18}$$

$$v_{t,i} \geq -(x_{t,i} - x_{t-1,i}) \tag{19}$$

$$x_{t,i}, z, p_{t,i}, s_{t,i}, v_{t,i} \geq 0 \; ; \forall i \tag{20}$$

$$y_{t,i} \in \{0,1\} \; ; \forall i \tag{21}$$

In the final model, $x_{t,i}$ is the weight of stock i in period t, t is the time period index in the range $t = 1, \dots, T$, $\mu_{t,i}$is the return of stock i in period t, μ_0 is the minimum expected return, $v_{t,i}$ is an auxiliary variable for transaction costs, $s_{t,i}$ is an auxiliary variable for Yager entropy, λ is the parameter representing the importance coefficient of the dual objectives (risk and entropy) in the problem's objective function, Γ is the tuning parameter for the uncertainty budget and the degree of the problem's uncertainty, $p_{t,i}$ and z are dual variables associated with robust optimization.

Findings and Results

In this section, we first report the numerical results of the optimal portfolios for each period by solving the proposed model of this research. Subsequently, by defining a benchmark model as a reference, we evaluate the performance of the proposed model in comparison to the benchmark model.

Given the multi-period structure of the problem in this research, the investment horizon is divided into three parts. In period zero (the initial period), an optimal portfolio is formed. Then, to improve its performance in line with market and stock price changes, through two consecutive periods and using the repeated optimization approach, a new optimal portfolio is formed by updating the available data for each stock, and its performance is evaluated using out-of-sample data.

4.1 Optimal Portfolios

As explained, to understand and evaluate the performance of this research's proposed model, we need to compare the obtained results with a benchmark or reference model. In the field of portfolio optimization, one of the most common and well-known benchmark models is the Equal-Weight Portfolio, first introduced by DeMiguel et al. (2009). This portfolio represents a simple yet highly effective strategy, and in the majority of cases, outperforming it is a very difficult task. This strategy, also known as the Naive Portfolio or $1/n$, allocates equal weight to all available stocks and often demonstrates significantly better performance than more complex models. In this research, this portfolio (strategy) is used to compare the performance of the proposed model. In the remainder of this research, we refer to our proposed model, fully introduced at the end of Chapter Three, as the "Robust Mean-Variance Entropy Model" or, abbreviated, GRMVE. Table 2 shows the optimal portfolio for each period.

Table 2. Optimal Portfolios by Period

Stock	GRMVE Model			Naive ($\frac{1}{n}$)
	Period 0	Period 1	Period 2	
NVDA	0.00000	0.00000	0.05000	0.05
MSFT	0.05000	0.05000	0.06061	0.05
AAPL	0.07558	0.07766	0.07545	0.05
GOOG	0.06621	0.06035	0.06064	0.05
AMZN	0.05000	0.05000	0.05885	0.05
META	0.05000	0.05000	0.05000	0.05
AVGO	0.05000	0.05000	0.04735	0.05
TSLA	0.00000	0.00000	0.00000	0.05
BRK-B	0.09832	0.09922	0.10372	0.05
WMT	0.09912	0.10217	0.08789	0.05
JPM	0.07336	0.07483	0.07464	0.05
ORCL	0.05000	0.05000	0.05137	0.05
V	0.08003	0.09992	0.09992	0.05
LLY	0.05000	0.05016	0.05016	0.05
MA	0.05000	0.05000	0.00000	0.05
NFLX	0.00000	0.00000	0.05000	0.05
XOM	0.08043	0.08569	0.00000	0.05
COST	0.07695	0.05000	0.07939	0.05
PLTR	0.00000	0.00000	0.00000	0.05
JNJ	0.00000	0.00000	0.00000	0.05

Figure 2 has been plotted to better visualize the results obtained in Table 2 and to illustrate the capital allocation among the selected stocks in the optimal portfolio for each period. By examining the structure of the portfolio in each period, one can observe the high diversity of stocks in each. On the other hand, the high similarity between the selected stocks and their optimal weights in the portfolio of one period compared to the next indicates the stability of the GRMVE model in the portfolio optimization process. The result of this stability is lower transaction costs due to smaller volumes of changes resulting from buying and selling stocks at the beginning of each period. The main changes made in the structure of the portfolio in the second period include the addition of stocks from companies NVDA and NFLX, and the removal of MA and XOM.

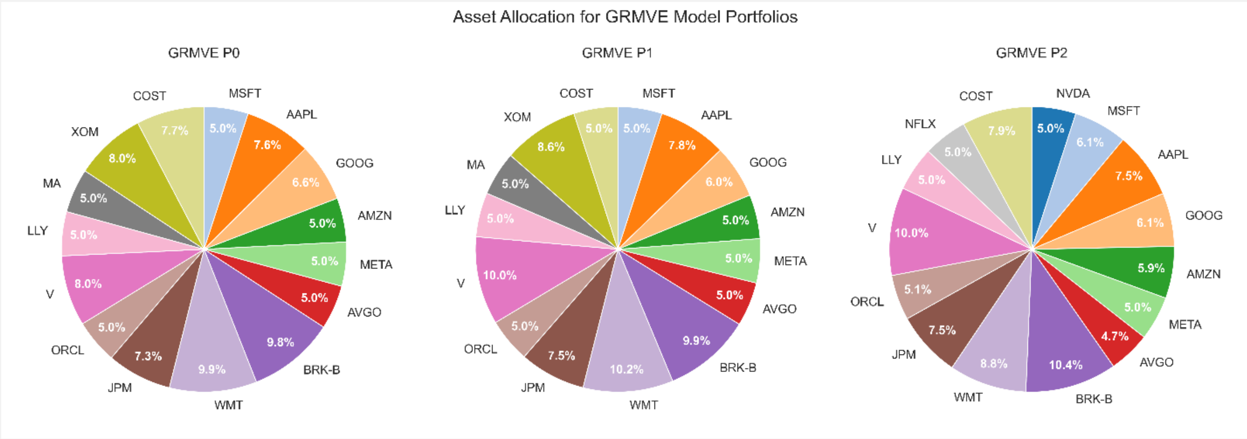

Figure 2. Asset Allocation of GRMVE Model

One of the important components incorporated into the mathematical model of this research is Yager Entropy as a diversification metric. A key point regarding the Equal-Weight $(1/n)$ portfolio is the maximum achievable diversification by this strategy. However, the main question can be posed as follows: To what extent is Yager entropy capable of creating diversification within the optimal portfolio structure? To answer this question, Figure 3 is plotted to compare the diversification level of the optimal portfolios resulting from the GRMVE model with the Equal-Weight $(1/n)$ portfolio, thereby enabling the evaluation of this diversification metric's performance.

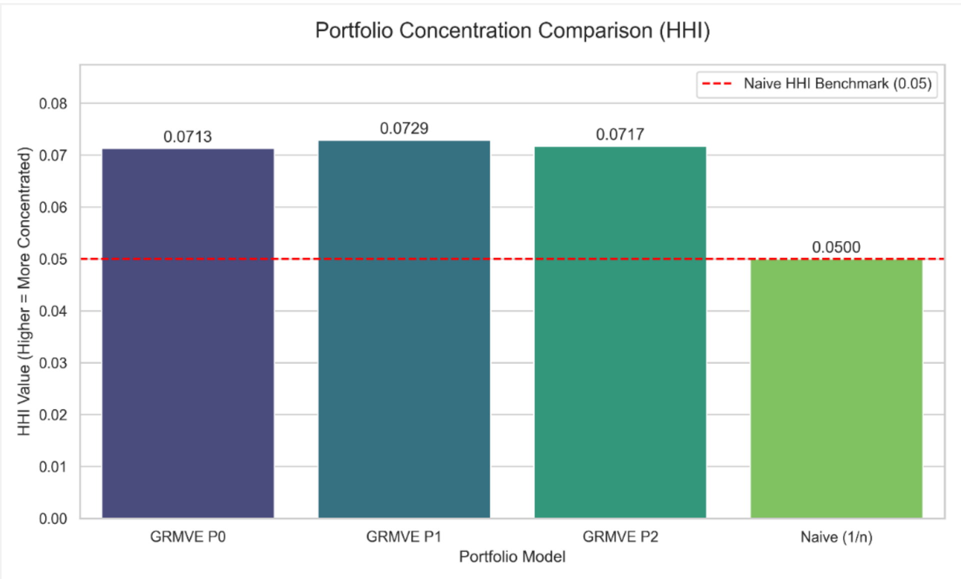

Figure 3. Portfolio concentration Comparison using HHI

The results displayed in Figure 3 are calculated using the HHI (Herfindahl-Hirschman Index). This index is a standard and widely used method for measuring the degree of diversification within a portfolio or the level of concentration in a portfolio or market. The obtained HHI index results lie within the range [1/n, 1]. Lower values indicate greater diversification, while higher HHI values signify investment concentration and allocated weight in a small number of stocks. An HHI index equal to 1 means the entire capital is allocated to a single stock, or in other words, a single-asset portfolio. The HHI index is calculated as follows:

$$HHI = \sum_{i=1}^{n} x_i^2 \tag{22}$$

Based on Figure 3 and the very close HHI values of the GRMVE model portfolios and the equal-weight portfolio, it can be concluded that Yager entropy performs excellently in reducing investment concentration, increasing portfolio diversification, and achieving results very close to the $1/n$ state.

4.2 Out-of-Sample Return and Performance

Following the examination and analysis of the structural characteristics of the GRMVE model in the previous section, this section aims to evaluate the performance of this research's proposed model on out-of-sample data. Table 3 presents the numerical results of each model's performance on the out-of-sample data, which forms the basis for the subsequent analyses.

Table 3. Out-of-Sample Performance

Month	GRMVE Model	)$\frac{1}{n}$(Naive
JAN	0.0487	0.043
FEB	0.0073	-0.0062
MARCH	-0.0657	-0.0724
APRIL	0.0117	0.0444
MAY	0.0648	0.0779
JUNE	0.0551	0.0529

Given the multi-period structure of the problem, the temporal division of data is as follows: the initial 12 months for period zero, and two consecutive 2-month periods for periods one and two. Consequently, the performance of each optimal portfolio has been recorded and reported over the 2 months following its formation.

During this 6-month interval, as per Table 4, the GRMVE model and the equal-weight portfolio achieved total returns of 12.19% and 13.96%, respectively. Despite the better overall performance of the equal-weight portfolio, a notable point is the manner in which this return was achieved and the trajectory followed by this strategy. According to Table 4, the number of negative return months for the equal-weight portfolio is 2, whereas the GRMVE model had a negative return only in the third month. This is highly significant from a risk management perspective and indicates the GRMVE model's ability to preserve capital under coverage.

To achieve a comprehensive comparison of the GRMVE model's performance with the equal-weight portfolio, this section introduces two widely used metrics that are highly effective for precise evaluation of these two approaches. If we wish to analyze the risk-return ratio of an investment, we can utilize two performance metrics: the Sharpe Ratio (Sharpe, 1966) and the Sortino Ratio (Sortino, 1994), which are among the most important and common performance measures. The Sharpe ratio measures the portfolio's excess return, the return achieved by the portfolio compared to the risk-free rate, per unit of risk. On the other hand, the Sortino ratio, as a more complete version of the Sharpe ratio, distinguishes between desirable and undesirable returns, considering only downside risk (undesirable returns) in its calculations. The Sharpe and Sortino ratios can be calculated using the following mathematical relations:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p} \tag{23}$$

$$Sortino\ Ratio = \frac{R_p - R_f}{\sigma_d} \tag{24}$$

In the formulas above, R_p is the portfolio return, R_f is the risk-free rate, σ_p is the standard deviation of excess returns, and σ_d is the standard deviation of downside returns (undesirable returns). Accordingly, the calculated values for each of these metrics for the GRMVE model and the equal-weight portfolio are shown in Table 4.

Table 4. Performance metrics for evaluation

Model	Sharpe Ratio	Sortino Ratio	HHI
GRMVE	1.3621	**N/A*	0.0720
Naive ($\frac{1}{n}$)	1.4174	2.1210	0.0500

The very close performance of the two models is clearly evident from the results in Table 4. A very important and interesting point regarding the GRMVE model, which was also discussed earlier, is its high capability in risk management and avoidance of negative returns. To calculate the Sortino ratio, the standard deviation of undesirable returns is placed in the denominator of Equation 24. However, given that the GRMVE model recorded only one negative return, its standard deviation of undesirable returns becomes zero (the standard deviation of a single number is zero), and consequently, its Sortino ratio is undefined (*N/A*).

To better illustrate the performance of the GRMVE model compared to the equal-weight portfolio on the out-of-sample data (6-month period), Figure 4 has been plotted. All the points raised regarding the higher volatility of the equal-weight portfolio's returns, the greater stability of the GRMVE model's performance, and its lower drawdowns (declines) compared to the equal-weight portfolio during downtrends are clearly observable in Figure 4.

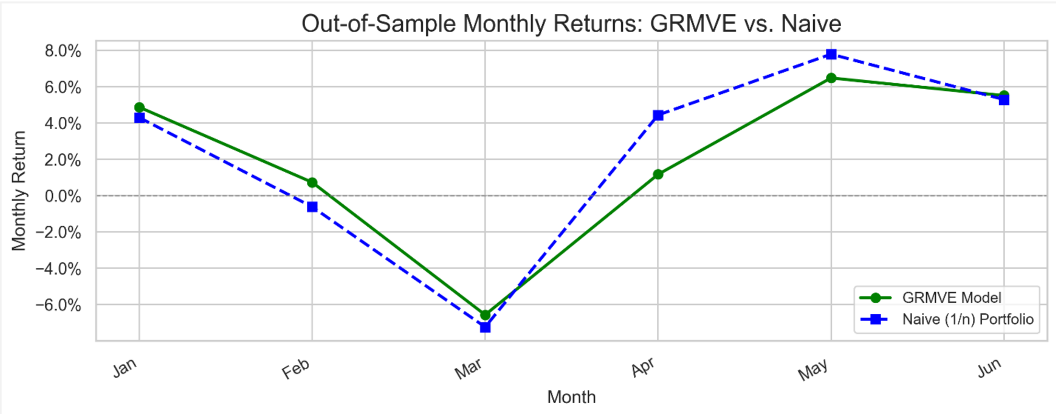

Figure 4. Performance Comparison of GRMVE and Naive Model

Discussion and Conclusion

The empirical findings of this study demonstrate that integrating robust optimization with an entropy-based diversification mechanism within a multi-period mean–variance framework produces portfolios that exhibit high stability, competitive returns, and superior downside protection relative to the naïve 1/N benchmark. Although the equally weighted portfolio slightly outperformed the proposed model in cumulative return over the six-month out-of-sample horizon, the robust mean–variance entropy structure achieved comparable performance while exhibiting fewer negative-return months and lower drawdown intensity. This result aligns with the well-documented resilience of naïve diversification reported in the literature (2), yet it also confirms that structurally enhanced optimization models can approach or match naïve performance when estimation risk and concentration effects are properly controlled. The Sharpe ratios of both models were close, suggesting that the robust-entropy design successfully mitigated the typical overfitting problem associated with classical mean–variance solutions. From a theoretical standpoint, this finding is consistent with the argument that the primary weakness of traditional optimization lies in error amplification rather than in the risk-return trade-off itself (3). By incorporating a budgeted uncertainty set, the model internalizes return estimation risk, thereby preventing extreme weight allocations that would otherwise emerge from small changes in input parameters (5, 6). The close alignment between the robust model's performance and the naïve strategy suggests that robustness can neutralize the structural advantage often attributed to equal weighting under noisy estimation environments (2).

A notable contribution of the results concerns diversification behavior. The Herfindahl–Hirschman Index (HHI) values of the robust entropy model were extremely close to those of the equal-weight benchmark, indicating that Yager entropy effectively prevents capital concentration without forcing mechanical uniformity. This observation corroborates the broader literature on entropy-based diversification measures, which emphasizes their ability to distribute weights more evenly while preserving optimization flexibility (9, 10). Recent advances integrating ordinal information and diversification constraints into portfolio optimization further support the importance of structured diversification to avoid corner solutions (11). In small and constrained markets, robust maximum diversified portfolios have demonstrated that diversification mechanisms combined with robustness improve stability and reduce vulnerability to structural shocks (8). The present findings extend these insights by showing that entropy-based diversification remains effective even when embedded within a multi-period robust framework under transaction costs. Importantly, diversification in this study did not significantly erode expected return, which addresses a common criticism that diversification penalties may dilute performance. Instead, the entropy component appears to function as a stabilizer, reinforcing portfolio balance across periods.

The multi-period repeated optimization structure also contributed meaningfully to performance stability. Portfolio weights exhibited relatively small changes between periods, implying reduced transaction volumes and lower implicit turnover risk. This finding aligns with prior research demonstrating that multi-period robust allocation strategies with transaction cost considerations yield smoother rebalancing paths (7, 16). Bibliometric evidence indicates that dynamic portfolio modeling has become central to modern optimization research, particularly in contexts requiring adaptability under evolving information (15). Scenario-based and multistage planning frameworks under budget uncertainty further show that dynamic adjustments enhance resilience when uncertainty is explicitly modeled (17). In the current study, repeated optimization allowed the model to update return estimates each period while preserving structural diversification, creating a balance between adaptability and stability. This dual property—dynamic responsiveness combined with controlled turnover—addresses a longstanding tension in portfolio management between flexibility and cost efficiency.

Another important dimension of the discussion concerns uncertainty modeling. By treating expected returns as uncertain parameters bounded within a budgeted uncertainty set, the model effectively guards against adverse deviations without resorting to excessive conservatism. The theoretical underpinnings of this approach stem from robust linear programming developments showing that uncertainty budgets control the degree of protection while preserving tractability (3, 6). The empirical stability observed in the out-of-sample phase suggests that moderate robustness, rather than extreme worst-case protection, is sufficient to reduce performance volatility. This observation is consistent with the argument that overly conservative uncertainty sets may unnecessarily sacrifice return potential (5). In comparison to stochastic or regret-based models, which incorporate volatility-sensitive risk measures (18), the budgeted robust approach maintains computational simplicity while delivering competitive performance. Moreover, robust modeling appears particularly valuable in high-volatility environments such as cryptocurrency markets, where parameter uncertainty is magnified (14). The findings therefore reinforce the conceptual claim that robustness acts as an antidote to estimation error amplification while preserving feasible solution structures.

The discussion would be incomplete without situating the results within the broader evolution of AI-enhanced portfolio management. Although the present study does not directly embed machine learning forecasting, it is compatible with predictive inputs generated by AI systems. Machine learning-driven return prediction models have demonstrated improved risk-adjusted allocation performance in digital asset markets (19), and time-series forecasting integrated with optimization frameworks enhances dynamic decision-making (20). Reinforcement learning approaches to portfolio ranking and matching similarly underscore the growing convergence between predictive analytics and optimization (21). AI-based evaluation frameworks emphasize performance assessment beyond variance metrics, incorporating broader criteria of stability and adaptability (4). The current robust entropy model can serve as a structural backbone for such AI-generated forecasts, ensuring that predictive insights do not reintroduce concentration risk or estimation instability. Furthermore, the ability to accommodate structural constraints such as ESG considerations demonstrates the extensibility of the framework to multi-criteria investment objectives (22). In this sense, the model represents not merely an incremental extension of mean–variance theory, but a modular platform adaptable to modern intelligent finance ecosystems.

Behavioral finance insights also provide interpretive depth to the results. Research on regret-based portfolio behavior suggests that investors tend to concentrate holdings in assets with recent strong performance, thereby increasing exposure to reversal risk (12, 13). By embedding entropy-based diversification, the model implicitly counteracts such behavioral biases, enforcing structural balance even when short-term signals might favor concentration. This design principle is particularly relevant in volatile equity markets dominated by high-capitalization stocks, where correlation clustering can amplify systemic risk. The stability observed in the HHI comparisons indicates that entropy regularization mitigates excessive reliance on a small subset of dominant firms. Hence, beyond technical robustness, the model offers a behavioral safeguard that tempers overreaction to recent performance patterns.

Overall, the findings demonstrate that combining budgeted robustness, entropy-based diversification, and repeated multi-period optimization yields portfolios that achieve a delicate equilibrium between return competitiveness and risk containment. While naïve diversification remains a formidable benchmark (2), the proposed structure narrows the performance gap while providing superior downside consistency. The synergy among robust theory (6), entropy-based diversification (11), and dynamic adjustment (15) appears to resolve key weaknesses of classical mean–variance optimization without incurring excessive complexity. These results reinforce the notion that future portfolio research should emphasize integrated frameworks rather than isolated methodological enhancements.

Despite its contributions, this study has several limitations. First, the empirical evaluation was conducted over an 18-month horizon using a selected subset of large-cap U.S. equities, which may limit generalizability across longer timeframes, different economic cycles, or alternative asset classes. Second, the uncertainty set calibration relied on a fixed budget parameter, and different tuning choices may produce varying trade-offs between conservatism and performance. Third, although the repeated optimization structure reduces turnover volatility, transaction costs were modeled proportionally and did not incorporate liquidity slippage or market impact effects. Finally, the study did not integrate explicit predictive machine learning models, meaning that return inputs were historical averages rather than forward-looking forecasts.

Future research may extend this framework in several directions. One avenue is the integration of advanced machine learning forecasting techniques within the robust entropy structure to assess whether predictive improvements translate into superior out-of-sample stability. Another direction involves testing alternative uncertainty sets, including ellipsoidal or distributionally robust formulations, to compare conservatism-performance trade-offs. Extending the model to alternative markets—such as emerging economies, cryptocurrency exchanges, or ESG-constrained universes—would enhance external validity. Additionally, exploring hybrid multi-period approaches that combine repeated optimization with partial rebalancing could provide deeper insights into turnover-efficiency trade-offs across different volatility regimes.

For practitioners, the results suggest that incorporating structured robustness and diversification mechanisms into portfolio construction can meaningfully enhance stability without sacrificing competitive returns. Asset managers may consider embedding entropy-based diversification penalties to avoid concentration risk in high-capitalization equities. Investment firms operating under uncertain macroeconomic conditions can benefit from moderate budgeted robustness to shield portfolios against estimation shocks. Finally, dynamic repeated optimization with disciplined turnover control offers a pragmatic pathway to balance adaptability and transaction efficiency in real-world portfolio management contexts.

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Authors’ Contributions

All authors equally contributed to this study.

Declaration of Interest

The authors of this article declared no conflict of interest.

Ethical Considerations

All ethical principles were adheried in conducting and writing this article.

Transparency of Data

In accordance with the principles of transparency and open research, we declare that all data and materials used in this study are available upon request.

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